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## **Cognitive Processes and Mental Models in Mathematics Teaching Using Artificial Intelligence**

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**Abstract:** In the context of increasing digitisation of education, this study examines the level of digital competence of recent high school graduates in the use of generative language models and equation editing tools in mathematics teaching. The research aimed to verify how effectively students can work with artificial intelligence tools in creating and solving word problems, primarily through systems of linear equations, and to identify the mental models they apply in doing so. The research was conducted on a sample of 25 first-year students from the Faculty of Education. The methodology employed a combination of quantitative and qualitative approaches, with the key analytical unit being the so-called semantic-logical structure (S-L structure), which enabled the



monitoring of the transformation of scientific content (A), the teaching task (B), and the student's mental schema (C). The results showed that students' digital skills are predominantly at the basic level – only 28% of participants were able to adjust the prompt and use AI effectively and critically. Most were satisfied with the output generated without more profound reflection. Using the equation editor was a completely new experience for more than half of them. The data also shows that while students can handle basic technical tasks, they often lack knowledge of more advanced tools, confidence, and tend to be satisfied with the first result without conducting a thorough check. On the other hand, it was confirmed that students who reflected on their mistakes and actively adjusted prompts were able to gradually improve their outcomes, which suggests that these skills can be developed in a targeted manner. The study highlights the need for the systematic development of digital literacy, problem-solving skills, and reflective thinking in future teachers. The S-L structure has proven to be an effective tool for monitoring cognitive processes in digitally oriented teaching and learning. Practical strategies include prompt engineering workshops, guided reflection, and subject-specific AI training to improve learning outcomes and e-learning quality.

**Keywords:** digital competence, generative language models, artificial intelligence (AI), semantic-logical structure (S-L structure), mathematical problems, systems of linear equations, equation editor, work with prompts.

## Introduction

High school graduates differ in digital competence when they enter college, and the success of college and future careers depends on it. Generative language models for word formation and interactive learning are an essential tool that is gaining importance in the academic environment. The presented study will examine how students can utilise these tools to solve mathematical and other word problems.

Artificial intelligence (AI) has represented a significant transformation of educational methodologies in recent decades, especially in the field of mathematics. The term artificial intelligence was coined by John McCarthy in 1956. That year, they organised the first conference on this topic in the USA (Hodges & Hofstadter, 2014). The use of AI has the potential to transform the way teaching and learning are conducted, with technologies such as virtual reality, augmented reality, and games capable of enhancing student learning and performance (Guishca Ayala et al., 2024; Li & Manzari, 2025). A significant benefit of AI is the ability to provide students with individualised and adaptive learning. This allows for a unique pace to be applied and adapted to each student's strengths and weaknesses (Zhang et al., 2025). At the same time, AI can evaluate student progress, highlight gaps in understanding, and offer feedback, leading to more effective learning (Li & Manzari, 2025; Mohamed et al., 2022). Gabriel et al. (2025) report that AI can play a key role in reducing math anxiety and provide students with the emotional and cognitive resources needed to regulate and balance the challenges associated with math learning. Studies have shown



that the use of AI improves students' academic performance in mathematics. AI applications have led to improved academic performance, while at the same time leading to positive student perceptions and higher levels of satisfaction (Cotič et al., 2024; Relmasira et al., 2023; Voskoglou & Salem, 2020; Intelligence & Neuroscience, 2023). AI-powered tools can specify tasks to promote cognitive engagement, helping individuals develop cognitive skills that lead to self-determined learning and critical thinking (Kaya & Yavuz, 2025). In parallel, there is an increase in students' interest in studying mathematics by making AI easier to solve problems and reducing the time it takes to complete assignments (Relmasira et al., 2023; Zhang et al., 2025). Interactive activities and immediate feedback contribute to student motivation. At the same time, AI offers support to teachers in structuring teaching, for example, by creating virtual worlds for students, thus providing quality education (Guishca Ayala et al., 2025). There will undoubtedly be significant changes in future classes. Research shows significant potential and benefits for teaching and learning mathematics (Voskoglou & Salem, 2020).

At the same time, it is essential to highlight the challenges and limitations associated with the use of AI and its implementation in the educational process. Studies highlight the high cost of technology and teachers' reluctance to introduce innovations into their teaching, as well as concerns about the potential replacement of teachers by technology (Zhang et al., 2025). At the same time, large language models (LLMs) such as GPT face problems with the verifiability of mathematical solutions and the accuracy of outputs, including so-called "hallucinations" of data. A significant concern is that increased AI literacy and trust in AI may lead to a greater reliance on AI tools, which has a significantly negative impact on key 21st-century skills, including problem-solving, critical thinking, creativity, collaboration, communication skills, and confidence (Zhang et al., 2025). The AI itself is faced with complex and analytical questions with a high cognitive load, and its accuracy decreases as the level of difficulty increases. AI does not encourage original thinking or common sense in problem-solving (Kaya & Yavuz, 2025). The following prerequisites for the successful integration of AI into math education are important for all actors in the process: teachers' ability to integrate technology into teaching (Li, 2025), AI literacy of teachers (Relmasira et al., 2023; Zhang et al., 2025). At the same time, teachers are not required to have programming ability, but to use AI responsibly and ethically. In this context, studies state that students and teachers, for example, confuse language models with search engines or attribute human characteristics to them. Another prerequisite for integration is the practical mastery of correct prompt input, which increases the quality of outputs. Li and Manzari (2025) surveyed 516 math teachers in China to examine the multiple influences on the use of AI in math teaching in primary schools. The purpose of the research was to improve student learning outcomes and optimise the educational experience for a deeper understanding and retention of mathematical concepts. The study concluded by supporting teachers in their professional development and community involvement. This approach is expected to lead to improved student learning outcomes (Li & Manzari, 2025). Awang et al. (2025) published research on current applications and the future direction of artificial intelligence (AI) in mathematics education. The aim was to help educators find the best AI tools and inform researchers about current



trends and other directions in this field. The result was a strategy for effectively integrating AI into mathematics teaching (Awang et al., 2025). In their study, Qui et al. (2022) publish the benefits of acquiring basic mathematical concepts using the internet and tools in the form of expanded game tactics using AI. Many countries are currently revising their curriculum and introducing AI-powered learning into their core documents (Hwang, 2022).

The research aims to examine the level of digital competence of high school graduates in using generative language models to create and solve mathematical problems. The results of this study should contribute to a deeper understanding of the relationship between digital competencies and academic success, providing evidence for reforming accredited curricula at higher education institutions.

### **Research Problem**

The research focuses on the following key questions:

1. What is the level of digital competence of secondary school graduates?
2. How do they use generative language models effectively?
3. What methods do they choose?

### **Digital Competence and the Transformation of Teaching in the Education System**

Integrating digital competences into the school education system is essential for modern educational strategies, but for many teaching staff in primary and secondary schools, it poses a significant challenge. This process requires teachers not only basic technical knowledge, but above all, the ability to effectively integrate these skills into various educational areas. According to the latest revisions of curricular documents, digital competences should be integrated into the entire scope of the Framework Education Program for Basic Education, the Framework Education Program for Secondary General Education and the Framework Education Program for Secondary Vocational Education. The use of presentations, online quizzes, educational applications, and generative language models (AI) can significantly enrich the teaching of languages, mathematics, science, and other disciplines. Therefore, teachers must be equipped not only with the necessary technical skills but also with methods for the effective use of these tools in teaching. The transition to digitally competent education is a gradual process that requires time, investment in resources and support at all levels of the education system. In addition to technical skills, it is essential to teach pupils to analyse information obtained from the Internet critically and to use digital media ethically.

### **Transition to Digitally Competent Education**

The methodological basis was a framework that was created in 2017 under the auspices of the European Commission, specifically the Joint Research Centre (JRC), which provides scientific and technical support for European policies – the European Framework for the Digital Competence of Educators: DigCompEdu. This framework was created in response to the need to support educators



in the effective use of digital technologies in teaching. It focuses on various areas such as pedagogical strategies, digital resources, collaboration and communication, and is designed for all levels of education, from primary schools to higher education environments. The aim is to develop the digital competences of educators at both a personal and professional level, enabling them to integrate digital technologies into the educational process and provide quality and inclusive education (Redecker & Punie, 2017). The DigCompEdu Educators' Digital Competence Framework serves as a model for these measures. The Czech school environment is notable for its unique approach, as it operates within the DigComp strategy outlined in its curricular documents, specifically the Digital Education Strategy (MŠMT ČR, 2014), which remained in effect until 2020. Still, at the same time, it emphasises the importance of employment and economic adaptability, which is, to a certain extent, a nonsensical approach (Černý, 2019). Experts agree that it is important to open education to new methods and ways of learning through digital technologies, to improve pupils' competencies in working with information and digital technologies, and to develop computational thinking of pupils at all types and levels of schools (Sharp, Coneway, & Diego-Medrano, 2016). The latest strategic document of the Czech education system is the Strategy for Education Policy of the Czech Republic until 2030+. The task of the Strategy 2030+ is to set the direction for the development of education and investment priorities for the next ten years (2020-2030+). Significant steps have also been taken in projects co-financed by the European Union, such as through the Operational Program Research, Development and Education (OP VV), which provides grants to schools and teachers to improve their digital skills (Edu.cz, 2024). A set of measures and key activities is currently underway that lead to the introduction of a significant revision of the Framework Education Program for Basic Education (Velké revize RVP ZV, 2024).

The main goal of the transition to digitally competent education is to equip pupils and students with digital competencies for the future world of work. The use of modern tools and applications is essential for achieving the educational goals outlined in the curriculum documents and for supporting current teaching methods. In the digitised world of the 21st century, it should be a matter of course that the pupil (student) can search, classify and critically evaluate information. They need to capitalise on the opportunities presented by the digital environment, while also being prepared for the risks associated with the use of digital technologies (Fryč et al., 2020).

### ***Educational area Mathematics and its application at primary and secondary schools***

Mathematics teaching using digital devices is currently being systematically introduced in primary and secondary schools, which include computer classrooms equipped with desktop PCs or iPads directly in the classrooms.

From the first grade, primary school students have access to the school domain, which allows them to use Microsoft 365 cloud services and communicate via school email. The school's online information system is commonly used for organising teaching. Pupils gradually acquire the skills necessary for independent use of these platforms and applications, including evaluating the



completeness of data, searching for information and creating simple tables and graphs in a spreadsheet. They work with data from various online sources and solve practical tasks using operating system tools or special applications on iPads (ŠVP ZŠ, 2024).

In secondary schools, math teaching is shifting further to include advanced digital tools and methodologies that allow students to perform complex analyses and calculations. Students are introduced to applications such as Mathematica and Maple for performing symbolic calculations and graphically representing mathematical functions. Statistics and data analysis courses include the use of software such as SPSS (Statistical Package for the Social Sciences) or the R programming language. Programming and algorithmic thinking are also an integral part of the curriculum, where students use languages such as Python or JavaScript. In addition to these tools, a generative language model (AI) is being introduced to allow students to create and analyse textual content within mathematical projects and research (ŠVP G, 2024).

Project-oriented learning enables students to apply mathematical skills to real-world problems, often involving the use of real-world data and mathematical modelling. Dynamic math software, such as GeoGebra, and modelling software, like Tinkercad or SketchUp, allow students to visualise better and understand geometric shapes, and develop creative and project skills to design and implement their own 3D models.

This described scenario represents the ideal state towards which the change in curricular documents is heading. However, this change is still in its infancy, and its successful implementation depends on many factors, including the conditions of individual schools, the qualifications and approach of the teaching staff, and the ability of teachers themselves to adapt and effectively apply new methods in their teaching. The proper integration of advanced digital tools and technologies into mathematics teaching requires not only a change in curricula, but also support for the education system, sufficient resources and continuous professional development of teachers. This integrated approach to digital education leads to the development of key digital competences that are essential for pupils' success in an increasingly digitalised world, preparing them for higher education and future careers.

### ***Development of Cognitive Processes in Mathematics Teaching***

Teaching mathematics in the educational process requires various cognitive processes, such as logical reasoning, information analysis and synthesis, and pattern recognition. In teaching, teachers must promote the development of these cognitive processes through exercises that require pupils to think logically, analyse problems, and explore different solutions. Each student creates their own mental models of mathematics to help them understand how mathematics works. However, it is not possible to influence pupils' mental processes in any other way than through interventions in the observed structure of teaching. From the teacher's point of view, the orchestration of teaching can therefore only be didactically grasped and managed based on observation of external manifestations (Janík et al., 2016, p. 222).



This research on the use of artificial intelligence (AI) in mathematics education can draw on the findings published by Gwo-Jen Hwang and Yun-Fang Tu (2021) in «Roles and Research Trends of Artificial Intelligence in Mathematics Education: A Bibliometric Mapping Analysis and Systematic Review». It provides deep insight into how AI can transform math teaching by personalising and streamlining learning processes. Hwang and Tu emphasise that AI helps to overcome traditional teaching methods by offering individualised learning strategies that can be tailored to each student's specific needs, leading to an increase in their mathematical competencies and interest in the subject.

### ***The Concept of a Semantic-Logical Structure Schema***

Semantic-logical structure (S-L structure) represents a methodology for organising and representing information with an emphasis on its semantic and logical levels. At the semantic level, it focuses on the meaning of individual elements and their relationships to broader conceptual units, thereby facilitating an understanding of the context and the content itself. This semantic dimension fosters natural associations between concepts, which is essential for effective communication and interpretation of data. The logical level then ensures the systematic arrangement of information according to hierarchical or other relationships, thus contributing to the consistency, uniqueness and internal cohesion of the structure (Allemang & Hendler, 2011).

The S-L structure connects these two levels – it assigns both meaning and logical links to each element, which connect the individual parts into a coherent and meaningful network (Kejriwal et al., 2021). This model is often used in the construction of semantic networks, where individual nodes represent concepts and edges between them express logical relationships. Visualisation using these components allows you to understand even complex conceptual units.

When examining educational transformations, which often take place intuitively in practice, the S-L structure is a useful analytical tool. Following the concept of schemes, as proposed by Rumelhart (2017), we utilise it as a means of conceptualisation and deeper understanding. In this study, we focus on the variability of the students' activity in solving tasks and analysing the answers, which allows us to identify specific patterns of behaviour and reactions to stimuli from the teacher.

We do not perceive S-L schemes as static models, but rather as dynamic memory and transformation nodes that are involved in intellectual and psychomotor operations during interactions with the environment and in problem solving itself (Derry, 1996). Schemes understood in this way form the basic structures enabling efficient information processing. The aim of the research is therefore to analyse and describe the influence of cognitive processes and didactic transformations on students' mental models, as well as how they manifest themselves in the way students think and solve assigned tasks (Kohout et al., 2023).



## **Methodika S-L structure**

Based on the above concept, a research methodology was developed that focuses on how students think and the mental models they use to solve specific tasks. It is based on the S-L methodology of the structure of scientific content, which has proven to be a tool for formal representation of knowledge, its analysis and work with mathematical concepts in the field of didactics of mathematics.

The content watched is viewed in three interconnected dimensions of the scientific scheme:

- Dimension A: determines the content of the lesson; comprehension is the goal of the lesson.
- Dimension B: thematisation of scientific content in the learning environment. It specifies the assigned task and communication related to solving the problem.
- Dimension C: represents the mental schemes of pupils when solving problems.

These three dimensions represent layers that are interconnected and whose deeper structure can be modelled using a semantic-logical scheme. Content can be understood simultaneously on three levels of existence – subjective, intersubjective and objective (Janík et al., 2020; Slavík et al., 2017). It is their connection that contributes to the overall integrity of teaching and enables a more effective response to the student's experience, pre-understanding, and motivation.

The S-L structure schemes thus offer a tool for monitoring educational transformation: from the initial structure of the professional content (dimension A), through the learning situation in which the student acquires new knowledge (dimension B), to the reconstruction of knowledge and its anchoring in the student's mental models (dimension C). They allow not only to notice changes in knowledge, but also in the way students process and apply this knowledge in practice.

The methodology also includes qualitative analysis, focusing on the reflection of students as they solve tasks in individual phases. This part of the research offers a deeper insight into the thought processes that students activate when solving tasks and observes how their approach changes during the educational process. Students' reflective statements help to reveal key moments that affect their progress and level of understanding.

The principle of transformation between individual dimensions can be illustrated graphically (see Image 1).

- Dimension A captures the original scientific model – a network of concepts and their relationships.
- Dimension B represents a didactically modified task to be solved, often supplemented by new or missing elements. Dashed lines in the diagram can represent these complements.

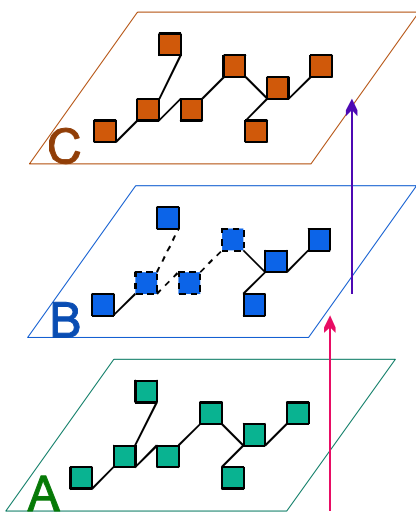


- Dimension C then displays the resulting structure that the student has created - ideally corresponding to the initial model and at the same time reflecting new understanding or consolidated knowledge.

In this way, the learning task becomes not only a tool for verifying knowledge, but also a means of active cognition, reflection and deeper understanding of the curriculum.

### Figure 1

*Principle of transformation in individual phases (a dashed line indicates the permissible limits of interpretative variability while maintaining the identity of the content)*



Source: Author's development.

### Research Methodology

The research was conducted in the winter semester of the academic year 2024/2025 at the Faculty of Education of the University of West Bohemia in Pilsen. It was aimed at first-year students in the Bachelor's degree program in Mathematics with an emphasis on education. The subject in which the survey took place is devoted to the introduction to the study of mathematics. In the first semester, the basics of secondary school mathematics are unified, as outlined in the grammar school curricular documents.

The students were selected from this group voluntarily. Of the 40 first-year students, 25 have signed up to participate. The ones mentioned expressed interest in participating in the research. The task itself was assigned during the first lecture, and the students were also introduced to the university's available hardware and software equipment, which they can use during their studies.

The research was conducted in accordance with the principles of voluntariness and transparency. The students were clearly informed about the task's goal, how the data would be processed, and how their anonymity would be ensured. At the same time, they had the option to terminate their participation at any time. Part of the measures was also that the success of their



solution had no impact on the evaluation during the semester. The students received feedback on their solutions only after the data collection was closed, to avoid affecting the results.

All activities were carried out in accordance with the ethical principles of the University of West Bohemia in Pilsen. Students were informed in detail in advance about the purpose of the task and the method of data processing. Participation was entirely voluntary, and anyone could unsubscribe at any time without any consequences. The collected outputs have been anonymised and used exclusively for this research. Feedback on the processed evidence was provided only after the end of the data collection, so that the progress of the task was not affected in any way.

### ***Tools and Procedure***

The data obtained were recorded in a table and further processed systematically, including sorting and categorisation, to identify patterns and topics related to the research questions according to the S-L structure methodology. First, the data was cleaned of unrelated data, and the structure of the columns and rows was standardised. Then the coding units were determined, to which codes reflecting their importance in relation to the research were assigned. Coding was done manually, and quantitative values were classified, creating a clear structure. Similar codes were then grouped into topics, which made it possible to reveal common patterns across the data (Hendl, 2023). Based on the data obtained, questions were formulated for a qualitative research study, which was conducted through interviews with students.

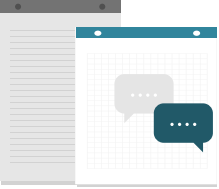
### ***Validity and Reliability of Data***

To ensure the highest possible reliability and credibility of the obtained data, the triangulation method was employed in the research. It consisted of a combination of several approaches - namely, the analysis of the S-L structure, group and individual interviews, as well as the analysis of documents. By combining these methods, it was possible to look at the phenomenon from multiple angles and thus obtain a complete and more accurate picture of what is really happening (Patton, 2015).

The primary focus on ensuring internal validity was achieved through the careful selection of participants and a clear definition of research variables. This reduced the risk as much as possible that the results would be distorted by ambiguities in terms of procedures themselves (Shenton, 2004).

External validity was supported by situating the results within a broader context and discussing their potential applications in other educational settings.

The reliability of the entire research process was also important. This was ensured mainly by the fact that all steps were carefully recorded and described so that they could be repeated at any time. Thanks to this, the results can be verified in the future or compared with other similar studies (Hendl, 2023).



## **Implementation of Research**

The research presented was carried out through the following sequences:

Sequence 1: Within the research team, fundamental research questions were defined, and documents necessary for the implementation of the research were prepared. At the same time, the research methodology was determined, the research group was determined, and the date of the research implementation was planned.

Sequence 2: The actual implementation of the research and the subsequent processing of the obtained research data took place. Part of this sequence involved conducting qualitative interviews with students, which provided a deeper insight into the research topic.

Sequence 3: Data validity analysis and evaluation were performed, and the research question was answered. The results were processed into a final report.

The research was conducted among first-year students of the Faculty of Education at the University of West Bohemia in Pilsen, with a focus on the field combination, with a primary emphasis on mathematics. A total of 25 students from this specific group were included in the research. The deadline for the implementation of the research has been set for the first hour of the semester of the academic year 2024/2025. To conduct the research, the students had access to a set of Google Workspace and Microsoft 365 services through their school accounts, which they utilised. Through these accounts, they first signed up for the ChatGPT and Microsoft Copilot language models. Using language models, they created a word problem solvable using a system of two linear equations. The research focused on the effectiveness of students' use of generative language models in creating word problems, their ability to work with prompts, and any progress they made in the second task when working with a different language model. The students were tasked not only with generating the problem, but also with solving it - for this purpose, they used the equation editor for typesetting. They focused on determining the procedure and the methodology for solving, and supplementing their calculations with notes. To solve the task, the students prepared a template in a text editor and stored it on cloud storage. The prepared files were submitted exclusively in electronic form.

After submitting all the papers, the research data were processed according to the established methodology of the S-L structure. The data obtained was recorded in a spreadsheet and then coded in a systematic process, involving sorting and categorising the data to identify patterns and topics relevant to the research questions. First, the data was cleaned of unrelated data, and the structure of the columns and rows was standardised. Then coding units were chosen, to which codes reflecting their importance in research were assigned. Coding was done manually, and quantitative values were classified, creating a clear structure. Similar codes were grouped into themes, which allowed the identification of common patterns across the table (Hendl, 2023).

After completing the coding, the formulas and relationships were analysed in the context of research questions and hypotheses, which led to the creation of a clear transformation level of



dimension C in problem-solving. The whole process is visualised in detail in the following text. Based on the data obtained, questions were formulated for a qualitative research study, which was conducted in the form of interviews with students (Strauss et al., 1990).

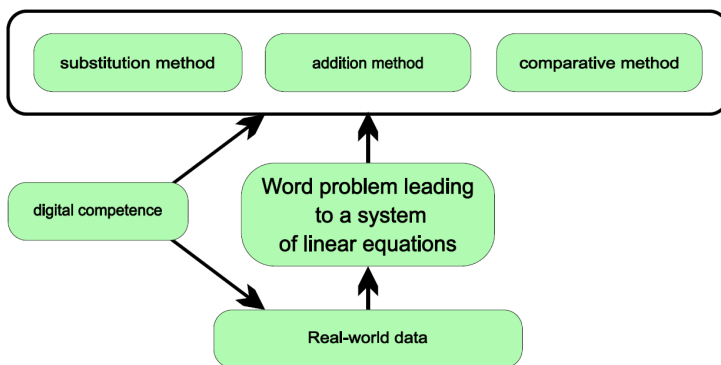
### Transformation Levels of Layers A, B, C, S-L Structures

Images 2, 3, 4, and 5 illustrate the differentiation of various layers. For easier orientation in the further explanation, we will name them. The level of the scientific schema (A), which is the source of concepts, will be called the concept layer. The level of the learning environment (B), in which scientific concepts are thematised in communication, will be called the thematic layer (Vosniadou, 1994). Finally, we distinguish the level of problem-solving (C), where the cognitive layer represents the characteristics of the pupil's mental scheme and the learning process.

### Scientific Scheme of Dimension 1

**Figure 2**

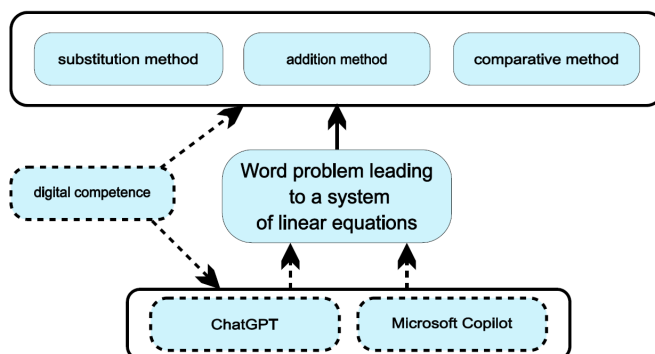
*Scientific scheme A - Word problems and their solution using a system of linear equations.*



Source: Author's development.

**Figure 3**

*Transformation level B - Creation of a word problem and its solution using methods for solving systems of linear equations.*

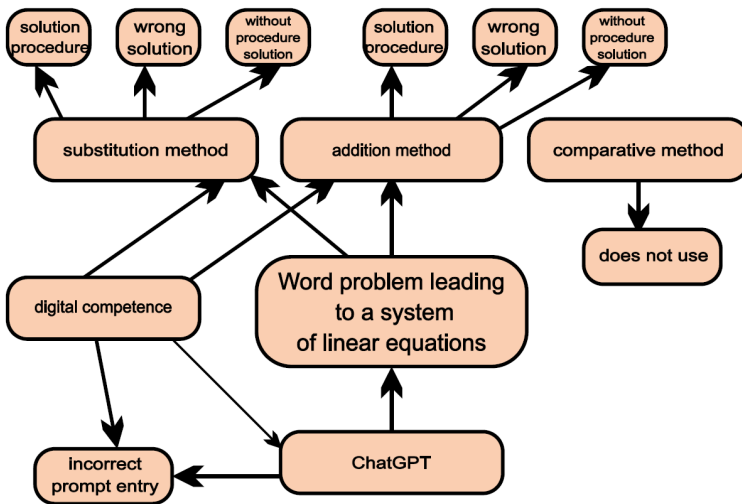


Source: Author's development.



**Figure 4**

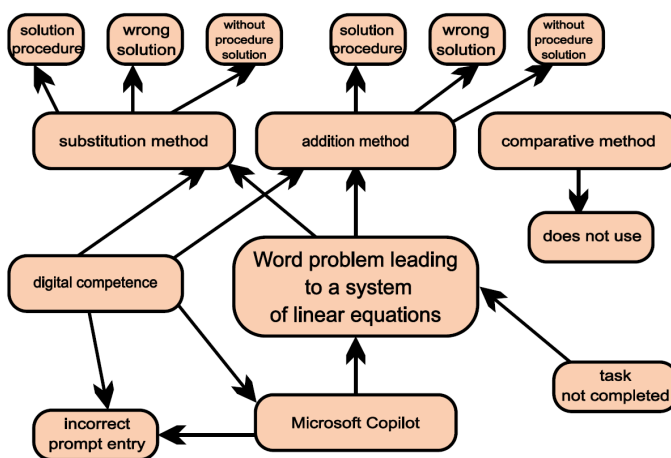
*Transformation level C1 - the network of the right solution to the first task using ChatGPT.*



Source: Author's development.

**Figure 5**

*Transformation level C2 - network the right solution to the second task using Microsoft Copilot.*

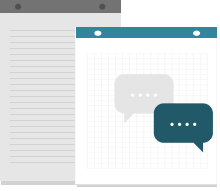


Source: Author's development.

### **Analysis of the C Transformation Level Using the S-L Structure**

The analysis of students' digital competencies was processed using an S-L structure, which enabled the capture of connections between their digital skills, the way they work when solving tasks, and how effectively they utilise generative models such as ChatGPT or Microsoft Copilot. This approach has made it easier to see how different factors – such as the level of digital competence, the chosen method of solving or working with prompts – affect results and their quality.

The results show that the digital competencies of secondary school graduates often remain only at the basic level. It turned out that one of the most significant problems is the inability to



critically and meaningfully revise prompt inputs, which is directly reflected in the quality of the tasks generated. This deficiency was also evident in the structure itself, where an element marked as "erroneous prompt inputs" repeatedly appeared. Furthermore, it turned out that students have limited experience using tools such as the equation editor, suggesting that they would need more support in this area as well.

As far as working with generative models is concerned, students most often created word problems that corresponded to classical solution methods, especially the substitution or addition method. However, due to poor work with prompts, tasks were often created that were too similar or even inappropriate. In addition, in the case of Microsoft Copilot, there were several unsubmitted assignments, which could be related to lower motivation or unclear assignments. On the other hand, it was confirmed that students who were able to use the experience from their previous work achieved better results, which shows how important regular practice is.

When solving the problems, students most often chose the addition and subtraction methods, while almost no one used the comparative method. This suggests that their knowledge may not be broad enough, or that they tend to stick to simpler practices. Working with the equation editor proved to be a weakness – many students admitted that they had never worked with this tool before, which was reflected in the quality of the submitted tasks and the fact that it took them more time to solve.

Overall, the S-L structure revealed that students' digital competencies are not yet at a level that would meet the demands of working with modern tools and generative models. The biggest weakness remains working with prompts and editing mathematical notations, which significantly affects students' results and motivation. These findings show that it is essential to support students more in working with modern technologies and to give them clear and comprehensible methodological guidance.

### ***Analysis of Key Students' Works with Focus Group Feedback***

The assessment focuses on a detailed evaluation of student work using feedback received from the focus group. The main objective is to examine the key factors that contributed to the development of this work, while identifying areas with potential for further improvement. Qualitative analysis of the documents and interpretation of feedback from the focus group reveal elements that positively influenced the acquisition of skills and knowledge (Krueger & Casey, 2015).

The evaluation of the use of generative language models, such as ChatGPT and Microsoft Copilot, in solving systems of linear equations focuses on a step-by-step analysis of working with these models in the context of creating and solving word problems that a system of linear equations can solve. The first step of the analysis is to investigate the generation of prompts and the subsequent formulation of a word problem that a system of linear equations can solve. Emphasis is placed on a specific procedure for solving the system using an equation editor and a methodological



approach to the solution. The next step is to compare the approaches of ChatGPT and Microsoft Copilot models in solving a similar task, with an emphasis on evaluating the differences in solution methods and their impact on students' thinking and perception of linear equations. The study focuses on how different approaches of generative models affect the students' understanding and processing of solutions to systems of linear equations. Qualitative analysis also includes interviews with the solvers, their observations, and perspectives, which provide a deeper insight into how students understand and approach different methods of solving problems of this type (Patton, 2015).

### ***Student theses: Task with the generative language model ChatGPT***

The first task in the paper was focused on the use of the generative language model ChatGPT. The students were asked to create a word problem that could be solved using artificial intelligence (AI) and a system of linear equations. Subsequently, they had to solve the generated task independently.

The resulting solution was to be processed using an equation editor with an emphasis on the proper formatting of the mathematical expressions. In addition, students were asked to add a comment that explained the key steps of the solution and provided a verbal response. This procedure included not only the technical processing of the task, but also the ability to use tools for creating and editing mathematical content effectively.

All students from the focus group worked on an assigned task and submitted it electronically for evaluation. Out of a group of 25 students, 7 were able to work with a prompt and get the required text of the word problem, which represents 28%. These students demonstrated the ability to tailor the prompt to match the exact requirements of the assignment.

On the other hand, 18 students (72%) used the chatbot only to write down the assignment directly without any modifications or customisation of the prompt. These students were content to generate any task according to the assignment without paying attention to the adaptation of the resulting text. This difference in approach indicates a different level of skill in working with prompts. It indicates areas in which methodological support for the effective use of generative language models would be appropriate.

### ***The Following Findings Emerged from the Students' Answers During the Interview:***

Researcher: "How did you enter prompts?"

Student 1: "Anything was enough for me, as long as I knew how to solve it."

This student did not emphasise the quality of the generated task, but the fact that he was able to solve the problem, regardless of the accuracy or specifics of the assignment.

Student 11: "I required the number of solutions to the problem."



The student focused on the requirement for a specific number of solutions, indicating that he was able to formulate a clear goal for the generated text; however, he did not adapt the prompt to create different types of tasks.

Student 7: "ChatGPT kept generating the same task. I couldn't get it to produce a different one, and then it started generating nonsense."

This student struggled with repetition and restriction of variation in the generated tasks, which indicates difficulties in formulating the prompt so that the model generates a broader range of tasks or more accurately matches its requirements.

This feedback from students shows different approaches and demands regarding working with a generative model.

### ***Analysis of the Solution of the Word Problem and Work with Mathematical Notation in the Equation Editor***

Out of the group of 25 students who worked on the given word problem, 10 students (40%) used the substitution method for solving, 13 students (52%) the addition method and 2 students (8%) used another way of solving equations. No one used the comparative method. A total of 19 students (76%) solved the problem correctly, but only 9 students (36%) used the equation editor. A comment on the procedure for the correct solution of a system of linear equations was recorded by 9 students (36%).

### ***The Following Findings Emerged from the Students' Answers During the Interview:***

Researcher: "What was decisive in choosing a method for solving a system of linear equations?"

Student 3: "I prefer the substitution method, but I also tried the addition method."

The student prefers the substitution method but is willing to experiment with another method, demonstrating a willingness to be flexible in mathematical approaches.

Student 5: "I had the problem solved using AI. I gave a similar solution procedure."

This student relies on an AI-generated solution, which indicates a tendency to utilise a generative model as an auxiliary tool and then adapt the procedure as needed.

Student 7: "They were simple problems; I don't care what method I use to solve them."

The student has no preference for the method, which may be due to their perception of the task as straightforward or to their overall lower involvement in choosing mathematical methods.

### ***The Following Findings Emerged from the Students' Answers During the Interview:***

Researcher: "How did you find working with the equation editor?"



Student 1: "It was completely new to me. I didn't encounter it in primary and secondary school. It took me a long time; I had to learn it. I would have it faster with written text."

For this student, working with the equation editor was a new experience, indicating that it required time to learn, which may point to the need for more intensive training in using this tool.

Student 13: "We only worked with the equation editor for one hour in elementary school, not in high school at all. I'm glad I learned it."

This student expressed satisfaction that he learned how to use the equation editor, which indicates a positive attitude towards new skills, despite minimal contact with the tool.

Student 18: I don't know that this function is in a text editor, nor that there are any rules for writing mathematical texts. I understand that I will have to learn it."

This student was unaware of the existence of an equation editor and typographic rules for mathematical texts, which points to a lack of awareness of the possibilities and standards for mathematical notation.

These interviews reveal varying levels of students' experience with the equation editor and methods for solving equations, highlighting areas for further educational support, particularly in working with mathematical notation in text editors.

### ***Student Theses: Task with Generative Language Model Microsoft Copilot***

The second task in the electronic document focused on utilising the generative language model Microsoft Copilot. The students had the same task as the previous one: to create a word problem using artificial intelligence that would be solved using a system of linear equations. Subsequently, they had to solve the generated task independently.

The resulting solution was to be developed using an equation editor with an emphasis on the proper formatting of the mathematical expressions. Students were asked to add a comment to the mathematical solution explaining the key steps of the procedure and to add a verbal answer. This task emphasised not only the technical processing of the task but also the students' ability to use tools for creating and editing mathematical content effectively.

To evaluate the solution, the researchers observed how the students applied their experience from the previous task, examining the extent to which they were able to utilise the acquired skills in the typography of a mathematical text and provide a precise description of the individual steps of the solution.

The students in the focus group worked on an assigned task and submitted it electronically for evaluation. Out of a group of 25 students who submitted a document electronically, only 19 students completed the second task; 6 students did not complete the work. Of the submitted papers, 6



students (32%) were able to work with a prompt and obtain the required text of the word problem. On the other hand, 13 students (68%) used the chat only to write down the assignment directly without any modifications or customisation of the prompt. These students were content to generate any task according to the assignment without paying attention to the adaptation of the resulting text.

The resulting solution was to be processed using an equation editor with an emphasis on the proper formatting of the mathematical expressions. Additionally, students were asked to include a comment that outlined the key steps of the solution and provide a verbal response. This procedure included not only the technical processing of the task, but also the ability to use tools for creating and editing mathematical content effectively. During the evaluation, the researchers took a view of the use of experience from the previous student task.

### ***The Following Findings Emerged from the Students' Answers During the Interview:***

Researcher: "How did you enter the prompts? How did you draw inspiration from the AI experience from the previous task?"

Student 3: "I tried to talk for a while and tell him what to write, but I wasn't very good at it. He kept forcing me to do one kind of task. I couldn't fix it."

The student attempted to communicate with the model and adapt the assignment, but was unable to achieve the desired result. He encountered repetition of similar tasks and was unsure how to adjust the prompt effectively.

Student 11: "I forgot to do the problem; I thought I didn't have to."

This student did not complete the assignment, which may indicate difficulties with remembering instructions or understanding the assignment's requirements.

Student 1: "I learned from the previous task; I tried different notations. I wasn't very successful. He was always writing something else. I don't want to do anything about it anymore."

The student tried to use the knowledge from the previous task and experimented with different prompt notations, but even so, she did not achieve a satisfactory result. Her frustration suggests that repeated failures with the model have reduced her motivation to continue.

Researchers' note: Student 1 was the only one who attempted to apply his previous experience to solve the next task. Although the student tried to experiment with different notations and apply the learned procedures, he still encountered difficulties, which led to frustration and a decrease in willingness to work with the generative model further.

These answers highlight the challenges students encounter when working with generative models, particularly in efficiently entering prompts and achieving desired outcomes.



### ***Analysis of the Solution of the Word Problem and Work with Mathematical Notation in the Equation Editor***

Of the 19 solvers who elaborated the given word problem, 10 students (53%) used the substitution method for the solution, 9 students (47%) the addition method, and none of the students chose any other way of solving systems of linear equations. A total of 16 students (76%) solved the problem correctly, but only 14 students (36%) used the equation editor. A comment on the procedure for solving a system of linear equations correctly was recorded by 9 students (47%).

#### ***The Following Findings Emerged from the Students' Answers During the Interview:***

Researcher: "What was decisive in choosing a method for solving a system of linear equations? Did the previous task affect the method of solving linear equations?"

Student 6: "I chose the substitution method. In the previous example, I chose the addition method. I wanted to use both ways."

The student actively tries different methods, which shows his interest in experimenting and gaining a broader understanding of individual approaches. Choosing different methods for various tasks demonstrates the deliberate application of acquired knowledge.

Student 5: "The previous solution inspired me; I copied the system from the previous problem and just changed the numbers."

This student prefers best practices and copies the method from the previous task. In this way, he avoids new approaches and instead relies on the efficiency of repetition, which gives him confidence in the correctness of his work.

Student 25: "I didn't care. I just wanted it to be right."

For this student, the correct result is crucial, while the method is less important to him. This pragmatic focus on the result suggests that he views methods merely as tools to achieve a correct outcome that has only one goal: to solve the problem correctly.

These answers demonstrate students' diverse approaches to solving problems: some students experiment with different methods and seek a deeper understanding, while others rely on established best practices to ensure correctness, and others focus solely on achieving a result.

The following table 1 is divided into categories used to analyse the results. The assigned codes (PT - text only, PP - modified the prompt, SM - substitute method, AM - addition method, JM - other solution method, A - yes, N - no, X - failed) correspond to the importance of individual units in the research, which made it possible to create a straightforward arrangement of the data. Coding was done manually, and quantitative values were systematically categorised, ensuring clarity for further analysis.

**Table 1**

*Coded table for analysis of research results (S - Student; GPT- Chat GPT; LR - LR systems; DC - digital competence; C - Correct; P - Progress; C - Copilot)*

S	GPT	LR	DC	C	P	C	LR	DC.	C	P
S 1	PT	AM	A	A	A	PP	AM	A	A	A
S 2	PT	SM	A	N	N	PT	SM	A	A	A
S 3	PT	SM	A	A	N	PT	SM	A	A	N
S 4	PP	AM	N	N	N	PT	AM	N	N	N
S 5	PT	SM	A	N	N	X	X	X	X	X
S 6	PT	AM	A	A	A	PT	SM	A	N	A
S 7	PT	AM	A	A	N	PP	SM	A	A	N
S 8	PT	SM	A	A	N	PT	SM	A	A	N
S 9	PT	AM	A	A	N	X	X	X	X	X
S 10	PT	AM	A	A	A	PT	AM	A	A	A
S 11	PP	AM	A	A	N	PP	SM	A	A	N
S 12	PT	AM	N	A	N	PT	AM	N	A	N
S 13	PP	AM	A	A	A	PP	AM	A	A	A
S 14	PT	AM	A	A	N	PT	AM	A	A	N
S 15	PP	SM	A	A	A	PP	SM	A	A	A
S 16	PT	SM	A	A	A	PT	AM	A	A	A
S 17	PP	AM	A	A	A	PP	SM	A	A	A
S 18	PT	SM	A	A	N	X	X	X	X	X
S 19	PP	JM	N	A	N	PT	X	X	X	X
S 20	PP	SM	A	A	N	X	X	X	X	X
S 21	PT	JM	A	A	N	PT	SM	A	A	N
S 22	PT	SM	N	A	A	PT	SM	N	A	A
S 23	PT	AM	N	N	N	X	X	X	X	X
S 24	PT	SM	A	A	N	X	X	X	X	X
S 25	PT	AM	A	A	A	PT	AM	A	A	A

Source: Author's development.

### **Statistical Analysis of Results with Confidence Intervals**

Despite the limited number of students, the results indicate that it is possible to estimate the range of similar results that would be obtained if the same task were assigned to a wider group. Indicative ranges, known as confidence intervals (95% CI), have been calculated for individual indicators, indicating the expected variance.

28% of students (7 out of 25) have successfully adjusted their work with a prompt from ChatGPT, with 95% certainty that the actual proportion in a similar group may be between 13% and 48%. At Copilot, the ability to work with prompts was at the 32% level (6 out of 19 students); the estimated interval is 15 to 55%. The correct resolution rates for both models were high, at around 76%, with a variance of 57 to 89% (ChatGPT) and 53 to 91% (Copilot). The use of the equation editor remained at approximately 36% among students, with an estimated range of 20% to 56%.



These data show that, although most students manage to solve the problem, working with a prompt or equation editor still causes difficulties for some of them. In addition, confidence intervals indicate that the results could vary significantly over a relatively wide range when repeated.

### ***Results and Conclusions of Action Research***

This chapter presents a detailed analysis of the results obtained during the implementation of the action research and their interpretation in relation to the study's objectives. The presented results are organised according to individual research questions and reflect the key processes that occurred during the implementation of the selected methodologies. The conclusions focus on identifying the benefits and limitations of the interventions and providing data for future applications in practice.

#### ***What Digital Competencies do Secondary School Graduates Have?***

High school graduates possess basic digital competencies, particularly in technical skills such as working with generative language models and utilising technology to solve mathematical problems. Most students mastered the assigned tasks, but only a third were able to work effectively with digital tools such as the equation editor. The key weakness is the limited ability to think critically and reflect, which is reflected in the inability to adapt the assignment to new situations. The analysis of the S-L structure reveals a strong correlation between digital competencies and difficulties with prompting or using tools, indicating a need for more targeted support in these areas.

#### ***How effectively can these students use generative language models to create word problems?***

The effectiveness of using generative language models was low. Only a small proportion of students were able to adapt the prompts to the assignment, resulting in frequent generation of low-quality or stereotypical tasks. Students generally struggled with transferring skills between tasks, which was reflected in their limited ability to apply learned procedures. The S-L structure emphasises the influence of incorrect prompt input and low motivation on the resulting work efficiency. Nevertheless, some students who reflected on their experience with previous tasks showed better outputs, indicating the possibility of gradual improvement in these competencies.

#### ***What Methods, along with Editing Mathematical Notations, will They Use to Solve These Problems?***

The most used methods were the supplementation and addition methods, while the comparative method remained unused. Most students were able to solve the problems correctly, indicating a solid understanding of basic mathematical procedures. However, the work with the equation editor was weak, as only a third of the students were able to use the tool effectively. The lack of experience with this tool led to a time-consuming process and frustration. The S-L structure confirms the link between method choice, output quality and digital competences, which points to the need for better preparation for working with digital tools in a mathematical context.



## ***Practical Application Implications***

Based on the research results on the digital competencies of secondary school graduates, it is necessary to strengthen support for digital literacy among both students and teachers. Research has shown that students have a weak ability to work with advanced digital tools and often mistype prompts, suggesting a need to revise school curricula and introduce adaptive technology tools that encourage individual learning paces and deeper engagement. It is also essential to develop critical thinking and problem-solving skills, to implement effective feedback and reflection in teaching, and to strengthen cooperation between schools and the IT sector, so that students can practically apply theoretical knowledge in real-world scenarios. These steps should help prepare students better for professional life in an increasingly digitised world.

## **Discussion**

### ***Possible Causes of Low Efficiency When Working with Digital Tools***

The research results indicate a significant discrepancy between the technical equipment of students and their ability to utilise modern digital tools in mathematics teaching effectively. Although most students have mastered the assignment from a technical perspective, their digital competencies are typically only at the level of an ordinary user. This finding aligns with the results of studies by Li and Manzari (2025) and Awang et al. (2025), which underscore the importance of systematically developing digital literacy in an environment of methodological support and guidance.

A fundamental shortcoming proved to be the students' limited ability to work effectively with prompts from generative language models. Teaching that focuses on creating quality assignments and critically evaluating outputs is still lacking. Our findings thus partially diverge from those of Zhang et al. (2025), who claim that quality prompting training leads to higher creativity and better outputs. In practice, students often generated stereotypical, repetitive tasks and did not utilise the possibility of modifying or repeating experiments.

Another weak point turned out to be working with the equation editor, which was a completely new experience for most students. Although they had regular Microsoft 365 tools at their disposal, technical tools were clearly not part of their previous school experience. This confirms the discrepancy between curricular objectives and the real state of teaching. As stated by Redecker and Punie (2017), the successful integration of digital technologies is contingent upon the availability of methodological support and the continuous professional development of teachers.

The students' answers indicate that they often view digital tools as a quick solution rather than a means to a deeper understanding of the curriculum. This pragmatic approach can weaken the ability to reflect on one's own work and lead to superficial solutions to tasks. Gabriel et al. (2025) note that without sufficient guidance, contact with more demanding technologies can lead to frustration rather than motivation - a finding confirmed in our research.



As far as specific mathematical methods are concerned, students preferred mainly filling and adding procedures, while they hardly used the comparative method. This may indicate both a limited repertoire of solving strategies and a tendency to stick to well-known and proven approaches. The results thus show the need to expand didactic strategies to help students develop a broader range of mathematical thinking.

### ***Critical Analysis of the Limitations of the Study***

It is essential to note that the research conclusions are based on a limited sample of students from one faculty and one year. Although methodological support was provided by triangulation (Patton, 2015) and the principles of validity were strictly adhered to (Shenton, 2004), the results cannot be generalised to the entire population of secondary school graduates.

Another critical factor is the context of the Czech education system itself, which is still in a phase of gradual digitisation (Fryč et al., 2020). Compared to education systems where AI tools are a standard part of the curriculum (e.g., China or Singapore - Hwang, 2022), barriers persist in the Czech Republic, particularly in the areas of technical equipment and teacher readiness (Černý, 2023).

From a methodological point of view, the research did not differentiate the level of students' previous digital experience. This can lead to distortion of results, especially if some students come from significantly different educational backgrounds, such as grammar schools or vocational schools.

We recommend following up with a longitudinal study to monitor the development of students' digital competences over a longer period of time. It could focus on the effect of systematic training in working with AI, the creation of quality prompts and the ability to type mathematical expressions correctly.

### **Conclusion**

The research confirmed that the digital competencies of secondary school graduates remain underdeveloped in their foundation. Although students can handle basic technical tasks, such as running an AI tool or using a mathematical model, they often struggle to reflect on the output critically, adapt the assignment using a prompt, or present a mathematical solution in the appropriate typographic form.

We consider the finding that most students encountered an equation editor for the first time at university to be a serious concern. This experience was often associated with frustration and time-consuming, which only underlines the need for its early integration into secondary education.

The inability to modify model outputs or detect erroneous assignment points is often attributed to the underdevelopment of critical thinking in the digital environment. Yet, this competence is



crucial for both the effective use of artificial intelligence and for future professional employment in a digitally oriented world.

However, the encouraging finding is that students who reflected on their mistakes and purposefully adjusted prompts achieved gradual improvement. This suggests that these skills can be effectively developed with appropriate didactic guidance.

### **Suggestions for Future Research**

- Introducing systematic training in working with generative language models and equation editors into teaching at secondary schools and universities.
- Develop didactic modules focused on creating quality prompts, correct typography of mathematical texts, and promoting reflective learning.
- To carry out follow-up longitudinal research with a wider and diverse sample of students to verify the impact of targeted digital skills development.

In conclusion, artificial intelligence in mathematics teaching offers considerable potential - if its use is conducted consciously, didactically, thoughtfully and with an emphasis on the development of critical thinking.

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